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Dual Electromagnetism and Magnetic Monopoles

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Dual electromagnetism (proposed some time ago) allows the fractional electric charges and the magnetic monopoles to exist simultaneously. In fact, the Dirac quantization condition can be numerically reduced (with some plausible assumptions) to the third component of the particle total weak isospin, which by definition is always quantized. The field angular momentum, **L**, of a static particle-magnetic monopole configuration is evaluated exactly; it is found that because the dual photon has a mass, M_c, **L** generally depends on r, the separation between a particle and a monopole. However, since $M_c \simeq 130$ GeV, at $r > M_c^{-1}$, **L** is basically dominated by ordinary electromagnetism and as such very weakly dependent on r.

Recently Barr et al. (1983) have introduced a "peculiar" photon into a grand unified model in order to reconcile the possible observations of fractional electric charges by La Rue et al. (1981) and of a magnetic monopole by Cabrera (1982). It may be pointed out that the idea of another photon is not all that new. Many people speculated about its existence at one time or another; some years ago this author introduced the dual photon within the framework of dual electromagnetic interactions (Šoln, 1979, 1980, 1981a, b). In this paper we wish to show how dual electromagnetism naturally allows the fractional electric charges and magnetic monopoles to exist simultaneously. Namely, the observed fractional charges (La Rue et al., 1981) appear in multiples of $q = e/3$, while the apparently observed magnetic monopole has the magnetic charge of $g = (2e)^{-1}$. This is clearly in contradiction with the original Dirac quantization condition (Dirac, 1931) $gq = n/2$, $n = 0, \pm 1, \pm 2, \ldots$. Hence, this condition has to be modified.

Let us start our discussion with the fact that flavor quantum numbers of basic fermions (leptons and quarks) as well as of other particles can be associated with the $SU(2) \times U(1)$ type of algebras (Šoln, 1979, 1981a). From this formalism we also deduced that to each flavor quantum number there

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corresponds a dual quantum number. Hence, in addition to the ordinary electric charge, there is the dual electric charge. This fact we took as an indication of the existence of dual electromagnetic interactions in nature. Subsequently we performed the unification of weak, electromagnetic, and dual electromagnetic interactions (Šoln, 1979, 1980). The Yang-Mills group which contains the electromagnetic, dual electromagnetic, and weak currents is clearly $SU_L(2) \times U(1) \times U'(1)$ (Šoln, 1980). A simple consistency requirement (Šoln, 1979, 1980) as well as the idea of the universality of the electromagnetic coupling constant in neutral currentlike interactions (Šoln, 1981b), suggests that dual electromagnetism and ordinary electromagnetism have the same strengths. This led the theory to a rather good agreement with experiments (Šoln, 1979, 1980). Within dual electromagnetism we have, of course, the dual photon. Its mass, M_c, unlike the mass of the ordinary photon, is not zero but about 130 GeV acquired through the spontaneous symmetry-breaking mechanism (Šoln, 1980).

In general, with respect to $U(1)$ and $U'(1)$, every particle carries electric charge $q = Qe$ and dual electric charge $q' = Q'e'$, respectively. Here e and e' are "elementary" electric and dual electric charges, respectively. [As already mentioned, numerically $e' = e$ (Šoln, 1980, 1981b).] Q and Q' are integral or fractional numbers satisfying

$$Q = I_3 + \tfrac{1}{2}(B - L) \tag{1a}$$

$$Q' = I_3 - \tfrac{1}{2}(B - L) \tag{1b}$$

where I_3 is the third component of the particle's total weak osospin, B its baryon number, and L its lepton number. The value of I_3 for a given particle one concludes from the $SU(2)$ representation assignments (Šoln, 1980, 1981a). For the basic fermions, I_3 can be read off from their two-dimensional assignments as follows:

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}, \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}, \ldots \tag{2a}$$

$$\begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}, \ldots . \tag{2b}$$

One sees (Šoln, 1981a) that

$$Q(e^-) = -1, \quad Q(\nu_e) = 0, \quad Q'(e^-) = 0, \quad Q'(\nu_e) = 1, \quad Q(u) = 2/3$$
$$Q(d) = -1/3, \quad Q'(u) = 1/3, \quad Q'(d) = -2/3$$

etc.

Let now a pointlike magnetic monopole have two kinds of magnetic charges also: g (magnetic charge) under $U(1)$ and g' (dual magnetic charge) under $U'(1)$. Such a pointlike monopole, which for simplicity is situated at the origin of coordinates, has magnetic and dual magnetic fields with the corresponding vector potentials (Coleman 1981/1982; Wu and Yang, 1976), as follows ($k = g, g'$):

$$\mathbf{B}_k = k\frac{\hat{r}}{r^2} \tag{3a}$$

$$\mathbf{A}_{(-),k} = \frac{\hat{\phi}k(1-\cos\theta)}{r\sin\theta} \tag{3b}$$

$$\mathbf{A}_{(+),k} = -\frac{\hat{\phi}k(1+\cos\theta)}{r\sin\theta} \tag{3c}$$

Potentials $\mathbf{A}_{(-),k}(\mathbf{A}_{(+),k})$ correspond to a monopole which has the Dirac string, at which the potentials are singular, extending to infinity along the negative (positive) z axis. According to Wu and Yang (Coleman, 1981/1982; Wu and Yang, 1976), for the singularity-free description of the monopole fields we should simultaneously utilize $\mathbf{A}_{(-)}$ and $\mathbf{A}_{(+)}$ vector potentials.

Now if we restrict ourselves to just one kind of vector potentials, say, $\mathbf{A}_{(-)}$, then a particle carrying charges of $q = Qe$ and $q' = Q'e'$ cannot detect the string via the Bohm-Aharanov effect (Coleman, 1982; Wu and Yang, 1976) if

$$M \equiv \frac{1}{4\pi}(Qe\Phi_g + Q'e'\Phi_{g'}) = n/2, \qquad n = 0, \pm1, \pm2, \ldots,$$
$$\Phi_g = 4\pi g, \qquad \Phi_{g'} = 4\pi g' \tag{4}$$

where Φ_g and $\Phi_{g'}$ are magnetic and dual magnetic fluxes through an infinitesimal circle encircling the string at any point on the negative z axis (Goldhaber, 1965). Relation (4) is now the modified Dirac quantization condition.

On the other hand, we may choose the singularity-free description of the monopole fields. Now, however, since $\mathbf{A}_{(-)}$'s and $\mathbf{A}_{(+)}$'s are connected through gauge transformations

$$\mathbf{A}_{(+),k} = \mathbf{A}_{(-),k} - \nabla\chi_k, \qquad \chi_k = 2k\phi, \qquad k = g, g'$$

the corresponding wave functions (Schrödinger, Dirac, etc.) $\psi_{(-)}$ and $\psi_{(+)}$, of a particle with charges Qe and $Q'e'$, also are related through the gauge transformation:

$$\psi_{(+)} = e^{-i2M\phi}\psi_{(-)} \tag{5}$$

The requirement that $\psi_{(+)}$ be single valued, assuming that $\psi_{(-)}$ already is, again yields quantization condition (4).

With the help of equations (1a, b) we have

$$M = eQg + e'Q'g' \tag{6a}$$

$$M = (eg + e'g')I_3 + (eg - e'g')\tfrac{1}{2}(B - L) \tag{6b}$$

Here one notices that M is symmetric if the roles of ordinary and dual electromagnetisms are interchanged: $e \leftrightarrow e'$, $Q \leftrightarrow Q'$, $g \leftrightarrow g'$ ($B \leftrightarrow -B$, $L \leftrightarrow -L$, $I_3 \leftrightarrow I_3$). As mentioned, comparison of the $SU_L(2) \times U(1) \times U'(1)$ gauge theory of weak, electromagnetic, and dual electromagnetic interactions with experiments suggests $e' = e$ (Šoln, 1979, 1980, 1981a, b). Furthermore, taking $g = 1/2e$ as suggested by Cabrera's experiment, we have from (6b)

$$M = \tfrac{1}{2}\left(1 + \frac{g'}{g}\right) I_3 + \tfrac{1}{2}\left(1 - \frac{g'}{g}\right) \tfrac{1}{2}(B - L) \tag{7}$$

This relation poses rather stringent conditions on g and g'. For example, $g' = -g$ is clearly not permitted, for it would not be possible to satisfy quantization condition (4) with $B = 1/3$. Also, we would have $M = 0$ for any meson, which would be rather unappealing from the physical point of view. Now we wish to argue that the choice of $g' = g$ is a natural one. Namely, if we take the invariance of M under the interchange of electromagnetism and dual electromagnetism seriously, then we have from (7) that $2 = g/g' + g'/g$ and $g'/g = g/g'$. These two equations are satisfied with $g' = g$ only. In fact, if in $eg = 1/2$ transformations $e \leftrightarrow e'$ and $g \leftrightarrow g'$ are performed and $e' = e$ is taken into account, then $g' = g$ again follows.

In any case for a pointlike magnetic monopole whose dual and ordinary magnetic charges are equal, $g' = g$, we have

$$M = I_3 \tag{8}$$

which not only satisfies quantization condition (4), but also specifies what n on the right side of (4) should be for every particle. As we see, the Dirac quantization condition appears to be associated with the third component of the particle total weak isospin. In fact, for a quantized theory (the quantized quantities appear with caps), the right side of equation (5) can be generated through

$$e^{i2\hat{I}_3\phi}\hat{\psi}_{(-)}\, e^{-i2\hat{I}_3\phi} = e^{-i2I_3\phi}\hat{\psi}_{(-)} \tag{9}$$

i.e., as a unitary "rotation" about the third axis of isospin. Here this axis can be formally associated with the geometric axis of the monopole string, since ϕ is an actual geometric angle. This axis also formally supplies the direction in the isotopic spin space along which isotopic spin **I** is diagonal. As we see, the magnetic monopole assigns geometric attributes to the isotopic spin space. However, relation (9) is not an isospin rotation because the generator is $2\hat{I}_3$ rather than just $\hat{I}_3$.

Let us now turn to the angular momentum, **L**, contained in the fields of a pointlike particle (with charges Qe and $Q'e'$) in the presence of a point magnetic monopole (with magnetic charges g and g'). The expression for **L** clearly is (compare with Jackson, 1975)

$$\begin{aligned} \mathbf{L} &= \frac{1}{4\pi}\int d^3x\, \mathbf{x}\times[\mathbf{E}_Q\times\mathbf{B}_g+\mathbf{E}_{Q'}\times\mathbf{B}_{g'}] \\ \mathbf{E}_Q &= \frac{Qe(\mathbf{x}-\mathbf{r})}{|\mathbf{x}-\mathbf{r}|^3}, \qquad \mathbf{E}_{Q'} = -\nabla(x)\,V_{Q'}(\mathbf{x}-\mathbf{r}) \\ \mathbf{B}_k &= \frac{k\mathbf{x}}{|\mathbf{x}|^3}, \qquad k = g, g' \end{aligned} \tag{10}$$

where the magnetic monopole, as before, is at the origin of the coordinates, and vector **r** denotes the position of the point particle. $V_{Q'}$ is the static potential of the particle due to dual electric charge $Q'e'$. From Šoln (1980) we easily deduce that the static potential has the Yukawa form:

$$V_{Q'} = Q'e'\frac{e^{-M_c|\mathbf{x}-\mathbf{r}|}}{|\mathbf{x}-\mathbf{r}|} \tag{11}$$

where $M_c \simeq 130$ GeV (Šoln, 1980). The range of this potential is clearly very short, about *mfm*. The evaluation of **L**, although tedious, can be done exactly. Here we just give the result:

$$\begin{aligned} \mathbf{L} &= -\hat{r}L, \qquad L = L_g + L_{g'} \\ L_g &= Qeg \\ L_{g'} &= 2Q'e'g'\{a^{-2} - e^{-a}[a^{-2}+a^{-1}]\} \\ a &\equiv rM_c, \qquad r = |\mathbf{r}| \end{aligned} \tag{12}$$

One verifies that $L_{g'} \to Q'e'g'$ when $a \to 0$ ($r \to 0$, M_c fixed; $M_c \to 0$, r fixed). As we see, unlike L_g, $L_{g'}$ depends on r. Hence, in general L and M are different [compare with equation (6a)]. Thus Dirac quantization condition (4) no longer implies that field angular momentum L is half-integer quantized (Saha, 1936, 1949; Wilson, 1949). The exception is at $r=0$, when $L(r=0)=M$. However, this is only of academic interest since at $r \lesssim M_c^{-1} \simeq$ *mfm* even leptons cease being pointlike.

For $r > M_c^{-1}$, L is dominated by L_g. This simply is the reflection of the fact that the dual electromagnetic interactions are only of about the *mfm* range. This also means that particles which carry both ordinary and dual electric charges (such as quarks, charged pions $\pi^{\pm}$, etc.) at distances larger than *mfm* basically will experience only the usual Coulomb's law. To summarize, while at distances larger than *mfm*, dual electromagnetism has no, or very little, effect on quantities such as forces, angular momenta, and the like, it nevertheless provides the correct Dirac quantization condition for the particle-monopole system, independent of their mutual distance.

Let us just mention that our analysis of the magnetic monopole did not require a color magnetic charge (t'Hooft, 1976; Preskill, to be published). (If it has a color magnetic charge, then it would also have to have a dual color magnetic charge whose fields would have to be screened at distances greater than *fm*.) Hence, the observed fractionally charged particle at Stanford could either be a quark or an isolated (colorless) particle.

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